ANALYSIS OF ELECTROMAGNETIC FIELD USING THE CONSTRAINED INTERPOLATION PROFILE METHOD

PHÂN TÍCH TRƯỜNG ĐIỆN TỪ SỬ DỤNG PHƯƠNG PHÁP CIP

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ABSTRACT

A new time domain method, CIP (Constrained Interpolation Profile) method has been developed as a numerical solver for multi-phase problems. The method is based on the upwind scheme for the finite difference method, but the variables to be calculated are not only the values of the quantity, but also the spatial derivatives. This study uses CIP method to analyze one-dimensional electromagnetic field and compare with the FDTD (Finite Difference Time Domain) method.

TÓM TẮT

Một phương pháp miền thời gian mới. Phương pháp CIP (Constrained Interpolation Profile) được phát triển để tính toán số trị các vấn đề đa pha. Phương pháp CIP không chỉ tính toán giá trị chuyển động của trường mà còn tính cả vi phân không gian. Trong nghiên cứu này sử dụng phương pháp CIP để phân tích trường điện từ một chiều và tiến hành so sánh với phương pháp Sai phân miền thời gian (FDTD: Finite Difference Time Domain).

Ⅰ. INTRODUCTION

Recently, time domain numerical analysis of electromagnetic (EM) fields has come to be investigated widely as a result of computer development. Development of accurate schemes is an important technical issue related to EM field calculation. Although many methods have been proposed, the finite-difference time-domain (FDTD) method is the most widely used method for time domain numerical analysis. However, the FDTD method solves Maxwell's equation using finite difference approximation. Using this approximation certainly causes error because of numerical dispersion. For that reason, EM field propagation cannot be analyzed exactly.

CIP method was proposed by Yabe.et.al. A noticeable feature of the CIP method is advantageous that it allows analysis not only of EM fields on grid points, but also analysis of their spatial derivatives on grid points.

In this study, CIP method is applied to one-dimensional EM field equation (Maxwell's equation) and compare analytical results with the FDTD method.

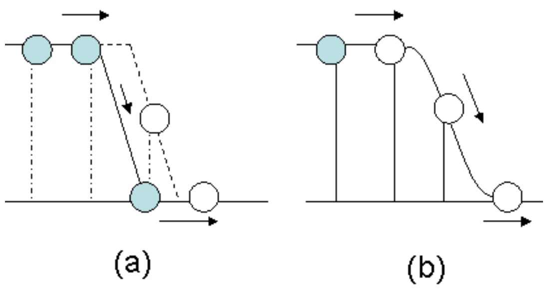

Fig. 1: The principle of the CIP method: (a) solid line is initial profile and dashed line is an exact solution after advection. (b) Spatial derivative also propagates and the profile inside a grid cell is retrieved.

Ⅱ. CALCULATION

A. The Constrained interpolation profile method

Let us consider the linear wave propagation in non-uniform mesh system with an advection equation:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = 0 \quad (1)$$

Then, let us differentiate Eq. (1) with spatial variable *x*, we get:

$$\frac{\partial g}{\partial t} + v\frac{\partial g}{\partial x} = -\frac{\partial v}{\partial x} g \quad (2)$$

Where *g* stands for the spatial derivative of f $(g = \frac{\partial f}{\partial x})$. In the simplest case where the velocity *v* is constant, Eq. (2) coincides with Eq. (1) and represents the propagation of spatial derivative with a velocity *v*.

If two values of *f* and *g* are given at two grid points, the profile between these points can be interpolated by cubic polynomial:

$$F_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \quad (3)$$

Here $F_i(x_i) = d_i = f_i$ (4)

$$\frac{dF_i(x_i)}{dx} = c_i = g_i \quad (5)$$

$$F_i(x_{i-1}) = -a_i\Delta x^3 + b_i\Delta x^2 - c_i\Delta x + d_i = f_{i-1} \quad (6)$$

$$\frac{dF_i(x_{i-1})}{dx} = 3a_i\Delta x^2 - 2b_i\Delta x + c_i = g_{i-1} \quad (7)$$

and $\Delta x = x_i - x_{i-1},\ D = -\Delta x$

From the conditions in the left-hand side the coefficients a_i and b_i can be described such as:

$$a_i = \frac{g_i + g_{i-1}}{D^2} + \frac{2(f_i - f_{i-1})}{D^3} \quad (8)$$

$$b_i = \frac{3(f_{i-1} - f_i)}{D^2} - \frac{2g_i + g_{i-1}}{D} \quad (9)$$

Thus, the profile at the n+1 step can be obtained by shifting the profile by $v\Delta t$ so that:

$f^{n+1} = F(x - v\Delta t), g^{n+1} = dF(x - v\Delta t)/dx$ and

$$f_i^{n+1} = a_i\xi^3 + b_i\xi^2 + g_i^n\xi + f_i^n$$
$$g_i^{n+1} = 3a_i\xi^2 + 2b_i\xi + g_i^n \quad (10)$$

Where $\xi = -v\Delta t$.

B. Electromagnetic Field Numerical Analysis Using the CIP Method

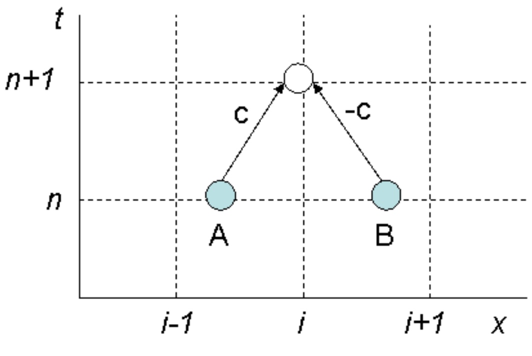

Fig. 2: Propagation of EM. Propagation of $B_z \mp E_y$ with velocity v is constant.

Faraday's and Ampere's laws are shown respectively as (11) and (12):

$$\frac{\partial B}{\partial t} = -c\nabla \times E \quad (11)$$

$$\nabla \times B = J + \frac{1}{c}\frac{\partial E}{\partial t} \quad (12)$$

Here, using CGS-Gauss system of unit and assuming $E = (0, E_y, 0), B = (0,0,B_z)$ and *J=0* in order to analyze one-dimensional EM field propagation of the *x*-direction in the vacuum, we can obtain the following equations from Eq. (11) and (12).

By addition and subtraction of Eq. (11) and (12), we obtion:

$$\frac{\partial B_z}{\partial t} = c\frac{\partial E_y}{\partial x} \quad (13); \qquad \frac{\partial E_y}{\partial t} = c\frac{\partial B_z}{\partial x} \quad (14)$$

In addition and subtraction of Eq. (13) and Eq. (14) for the derivative are also given as:

$$\frac{\partial(B_z+E_y)}{\partial t}-c\frac{\partial(B_z+E_y)}{\partial x}=0 \quad (15); \qquad \frac{\partial(B_z-E_y)}{\partial t}+c\frac{\partial(B_z-E_y)}{\partial x}=0 \quad (16)$$

It is readily apparent that equations from (13) to (16) are advection equations of $B_z \pm E_y$ and $\partial_x B_z \pm \partial_x E_y$. Therefore, application of the CIP method to these equations allows a solution of EM field propagation in the *x*-direction at time step *n*.

Next, we calculate the fields of the *n+1* time step from the fields of *n* time step. Here, the calculation method of the $\pm x$-direction propagation is described below using Fig. 2. Eq. (16) shows propagation of $B_z - E_y$ to *i* point with the velocity *+c* is constant. In Fig.2 shows propagation of $B_z^+ - E_y^+$ from A point to *i* point is become $B_z^{n+1} - E_y^{n+1}$. Eq. (15) shows propagation of $B_z + E_y$ to *i* point with the velocity *–c* is constant. Fig.2 shows propagation of $B_z^- + E_y^-$ from B point to *i* point is become $B_z^{n+1} + E_y^{n+1}$. We can obtion:

$$B_z^{n+1} - E_y^{n+1} = B_z^+ - E_y^+ \quad (17); \qquad B_z^{n+1} + E_y^{n+1} = B_z^- + E_y^- \quad (18)$$

In the *x*-direction, equations for the derivative are given as:

$$\frac{\partial B_z^{n+1}}{\partial x}-\frac{\partial E_y^{n+1}}{\partial x}=\frac{\partial B_z^+}{\partial x}-\frac{\partial E_y^+}{\partial x} \quad (19); \qquad \frac{\partial B_z^{n+1}}{\partial x}+\frac{\partial E_y^{n+1}}{\partial x}=\frac{\partial B_z^-}{\partial x}+\frac{\partial E_y^-}{\partial x} \quad (20)$$

Then, by addition and subtraction of these equations, we obtain:

$$B_z^{n+1}=\frac{(B_z^-+E_y^-)+(B_z^+-E_y^+)}{2} \quad (21); \qquad \partial_x B_z^{n+1}=\frac{(\partial_x B_z^-+\partial_x E_y^-)+(\partial_x B_z^+-\partial_x E_y^+)}{2} \quad (22)$$

$$E_y^{n+1}=\frac{(B_z^-+E_y^-)-(B_z^+-E_y^+)}{2} \quad (23); \qquad \partial_x E_y^{n+1}=\frac{(\partial_x B_z^-+\partial_x E_y^-)-(\partial_x B_z^+-\partial_x E_y^+)}{2} \quad (24)$$

Using the equations from (21) to (24), one can obtain EM field components of time step *n+1*.

From the above equations, $\partial_x = 1/\partial x$

Ⅲ. RESULTS AND DISCUSSION

We show the result of calculations in Section Ⅱ. We calculate the EM field propagation in the one-dimensional analysis domain in the vacuum. In the original addition:

$$E_y(x)=\begin{cases}1 & if \quad 0.4 \le x \le 0.6 \\ 0 & \quad otherwise\end{cases} \quad \text{and } B_z(x)=0$$

Grid size, $dx = 0.005m$; time step, $dt = 0.001s$; number of grid points, *NX*=200. Fig. 3 and Fig. 4 shows the electric field waveform that is calculated using CIP method and Fig.5 and Fig. 6 shows the magnetic field waveform that is calculated using FDTD method at $t = 0.0s, t = 0.05s$ and $t = 0.25s$.

All parameters of FDTD calculation are equal to those of the CIP calculation. Fig. 3, Fig. 4, Fig. 5 and Fig. 6 demonstrates that the CIP results provides accuracy. On the other hand, numerical oscillation appears in the FDTD result.

Next, we calculate the EM field propagation with the original addition using Gauss's distribution:

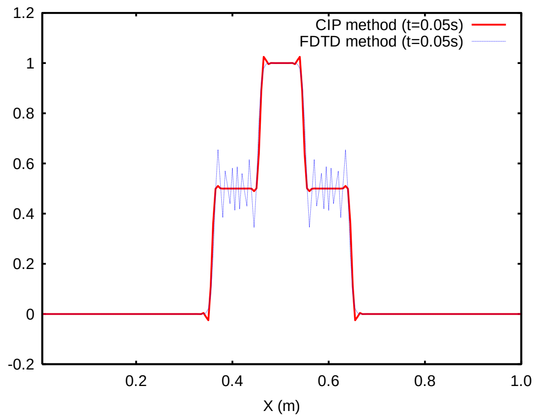

Fig. 3: Calculated results of Ey at t = 0.05s.

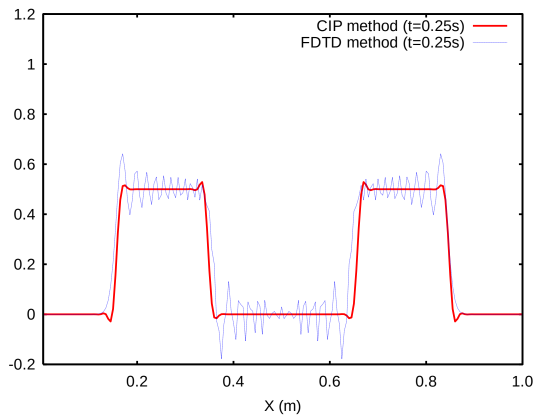

Fig. 4: Calculated results of Ey at t = 0.25s

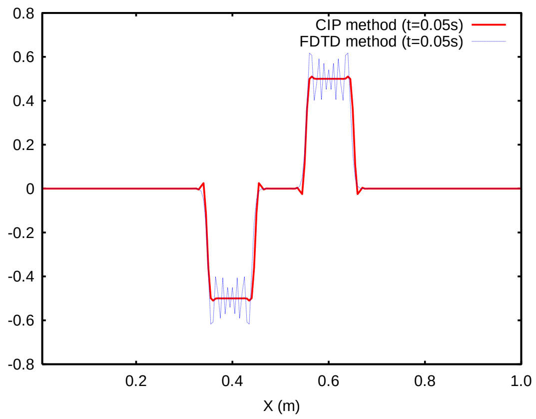

Fig. 5: Calculated results of Bz at t = 0.05s.

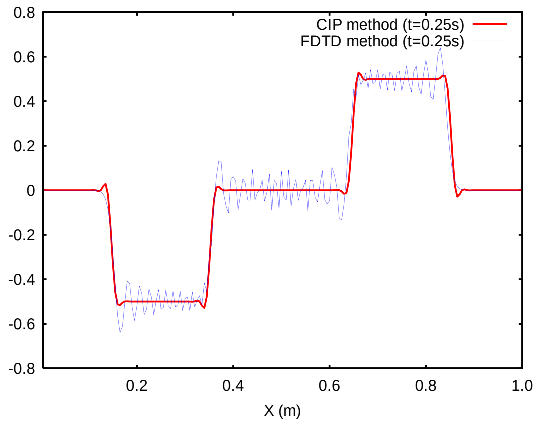

Fig. 6: Calculaed results of Bz at t = 0.25s

$$E_y(x) = 1 + 0.5\exp\left\{-\left(\frac{x-50}{\sigma}\right)^2\right\}, \qquad B_z(x) = 0$$

Parameters of the calculation are as follows: Grid size, $dx = 0.002m$; time step, $dt = 0.01s$; number of grid points, NX=200.

Fig. 7, Fig. 8 and Fig. 9 shows the electric field waveform that are calculated using CIP analysis and FDTD analysis under the condition of $\sigma = 5.0$, $\sigma = 1.5$ and $\sigma = 0.05$ at t = *0.6s*. When $\sigma = 5.0$, the results calculated using FDTD are equal to those of the CIP calculation. However, $\sigma = 1.5$ and $\sigma = 0.05$ numerical oscillation appears in the FDTD result.

Ⅲ. CONCLUSION

In this paper, we have analyzed EM field using the CIP method. We consider the values not only of the EM field, but also of their spatial derivatives. The results obtained in this study have clarified that EM field analysis using the CIP method provides higher accuracy than that obtainable using the FDTD method.

According to these results, we intend to investigate actual one-dimensional analysis including boundaries in the near future.

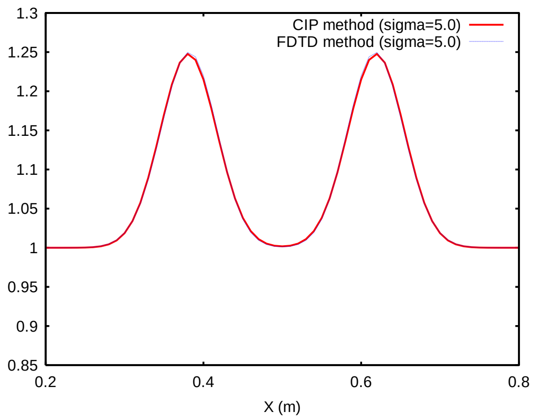

Fig. 7: Calculated results of Ey at $\sigma = 5.0$

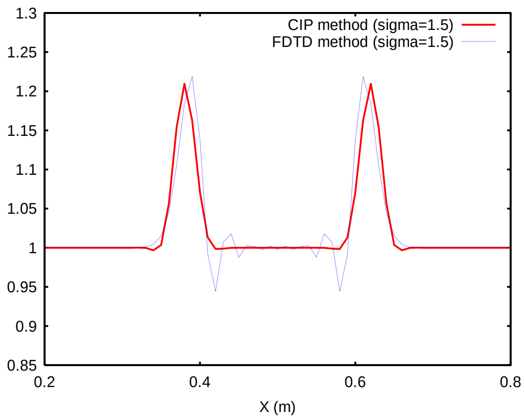

Fig. 8: Calculated results of Ey at

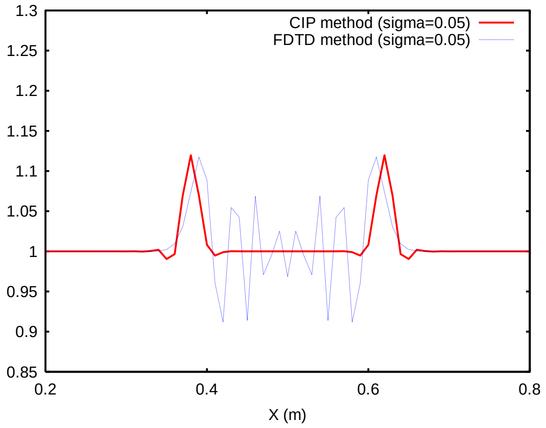

Fig. 9: Calculated results of Ey at

REFERENCES

[1] T. Yabe, T. Utsumi, and Y. Ogata, *The Constrained Interpolation Profile Method*. Morikita Publishing, Tokyo, (2003). (in Japanese)

[2] T. Yabe, X. Feng, and T. Utsumi, "The constrained interpolation profile method for multiphase analysis", *J. Comput. Phys*., vol. 169, pp.556-593, 2001.

[3] H. Takewaki, A. Nishiguchi, and T. Yabe, "Cubic interpolated pseudo-particle method (CIP) for solving hyperbolic-type equations", *J. Comput*. Phys., vol. 61, pp.261-268, 1985.

[4] T. Yabe, Y. Ogata, and K. Takisawa, *CG Simulation by CIP Method and Java*, Morikita Publishing, Tokyo, (2007). (in Japanese)

[5] K. Okubo, N. Takeuchi, "Analysis of an Electromagnetic Field Created by Line Current Using Constrained Interpolation Profile Method", *IEEE Trans. Antennas Propag*., vol. 55, no. 1, pp. 111-119, Jan. 2007.

[6] K. Okubo, Y. Yoshida and N. Takeuchi, "Consideration of Treatment of the Boundary Between Different Media in Electromagnetic Field Analysis Using Constrained Interpolation Profile Method", *IEEE Trans. Antennas Propag*., vol. 55, no. 2, pp. 485-489, Feb. 2007.

[7] S. Watanabe and O. Hashimoto, "An Examination of Electromagnetic Field Using CIP Method and Characteristic Curve Methods", EuMC 2005. (in Japanese)
[8] K. Okubo, N. Takeuchi, "Numerical Analysis of Electromagnetic Field Generated by Line Currendt Using the CIP Method", MW 2004. (in Japanese)

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